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Shape Optimization with Buckling and Stress Constraints

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Introduction

SHAPE optimization of stamped sheet metal parts has been the subject of much research.[1] All of this work has been based on static finite element analysis, which, therefore, permits only the consideration of stress and displacement constraints within the optimization problem. Although several papers exist on optimal sizing variables subject to buckling constraints, only one paper could be found dealing with shape variables.[2] That work addresses the problem of finding the optimal thickness distribution for a rectangular isotropic plate of given plan dimensions (length and width) that would maximize its uniaxial buckling load, loosely referred to as shape optimization. Other similar papers exist on such subjects as the design of stiffeners to maximize the buckling load of plates. The problem to be treated in this paper is the design of the shape of plates or curved shells of arbitrary shape to control the onset of global buckling, defined as follows:

$$[K]\{\phi_n\} + \lambda_n[K_d]\{\phi_n\} = 0$$

in which ϕ_n are the buckling modes, $[K]$ is the global stiffness matrix, $[K_d]$ is the differential stiffness matrix, and λ_n is the buckling load factor. If λ_n is ≤ 1.0, buckling is predicted.

Design variables control the shape of boundaries, holes, and cutouts. Internal mesh points move as the boundaries move using the static displacements of an auxiliary model.[3] A parametric modeling approach is used to define the complete problem, which includes fully automatic mesh generation[4] to create the finite element mesh. A commercial finite element program was used for analysis and sensitivities.[5] A conservative approximate problem formulation[6] is used for optimization in which the approximate problem is solved using the modified method or feasible directions.[7]

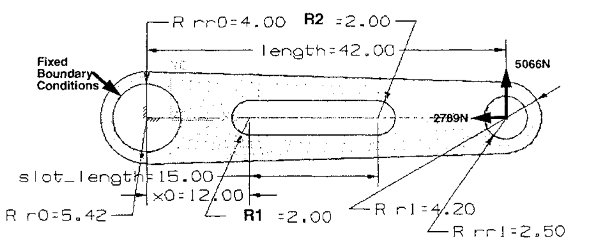

Fig. 1 Parametric torque arm mode (initial design): $E = 60E + 6$ N/cm^2; $\sigma_y = 80{,}000$ N/cm^2; $\rho = 0.00781$ kg/cm^3.

Design Problems

The following problems are designed for minimum mass, i.e., objective function, subject to constraints on the first buckling mode and the maximum effective stress in each element. There is one auxiliary load case per design variable. The analysis uses a four-node (quadrilateral), isoparametric membrane-bending plate element.

Rear Suspension Torque Arm

Analysis Problem

The component shown in Fig. 1 is a rear suspension torque arm, used to prevent windup in live axles. This problem is idealized as fixed around the large hole and loaded, as shown, at the small hole. The part is 0.3 cm thick and assumed to be stamped from sheet metal. For this study, the stresses are assumed to be limited to 80,000 N/cm^2.

Because this is a flat part loaded in the plane, without consideration of buckling, and even though no out-of-plane support exists (except at loading and support points), the only deformations expected would also be in the plane. However, the buckling phenomenon can also occur but is generally ignored. Linear static analysis will not detect buckling, which can be predicted by a separate buckling analysis. For the initial design of the torque arm, the first (lowest) buckling mode can be seen in Fig. 2. This figure shows the highly exaggerated out-of-plane deformation. Buckling occurs first in this

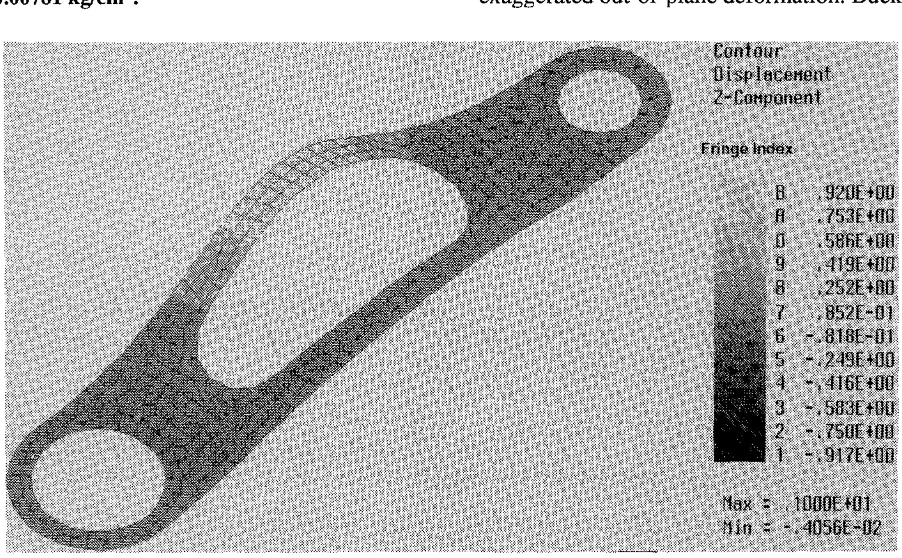

Fig. 2 Buckling mode of torque arm.

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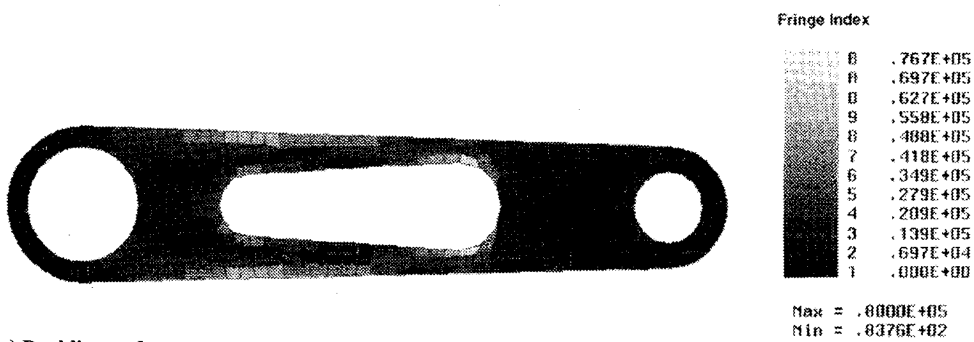
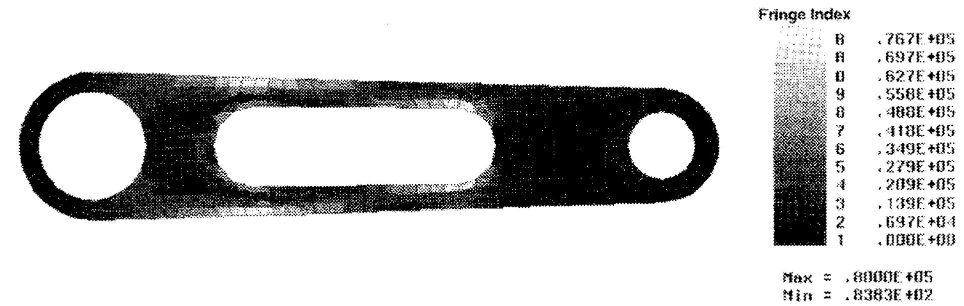


a) Buckling and stress



b) Stress only

Fig. 3 Final designs.

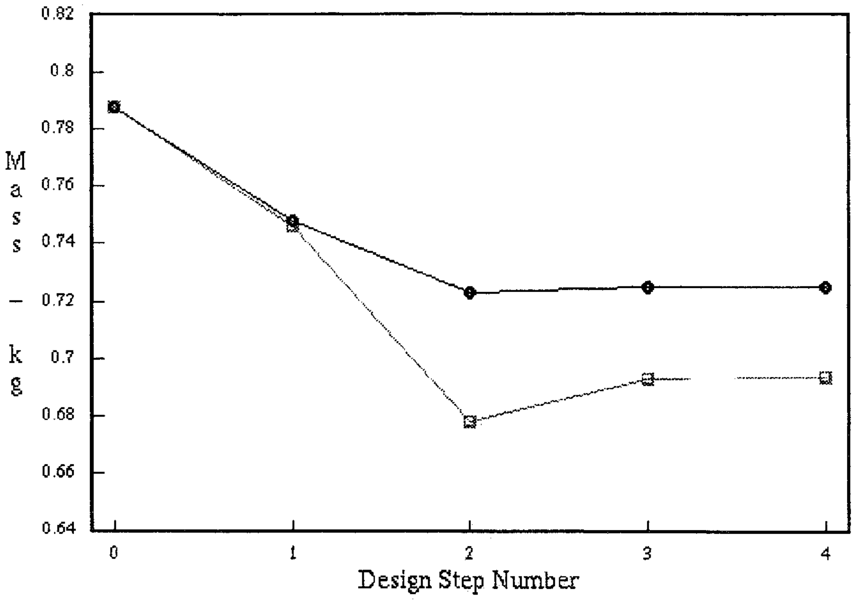


Fig. 4 Design history of torque arm: —⊟—, stress only; and —⊖—, buckling.

particular segment (top) of the slot as a result of the combination of applied loads in which the resulting stresses are compressive. In the other (bottom) slot segment, the bending load would cause tensile stresses.

Design Problem

The design problem is to minimize the mass of the part, which has two design variables, R_1 and R_2, as shown in Fig. 1. The linear elastic stresses are to be maintained at no more than 80,000 N/cm^2, and the part should not be allowed to buckle. The buckling mode shown in Fig. 2 for the initial design (2., 2.) has an eigenvalue of 1.08, which means that buckling will not occur. An eigenvalue of 1.0 or less indicates that buckling will occur. As the mass of the part is decreased by increasing the size of the slot, the critical buckling

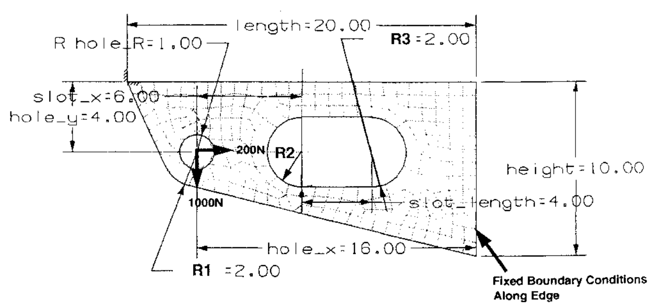


Fig. 5 Parametric model of brake bracket (initial design): $E = 1.3E + 6$ N/cm^3; $\sigma_y = 7500$ N/cm^3; $\rho = 0.00781$ kg/cm^3.

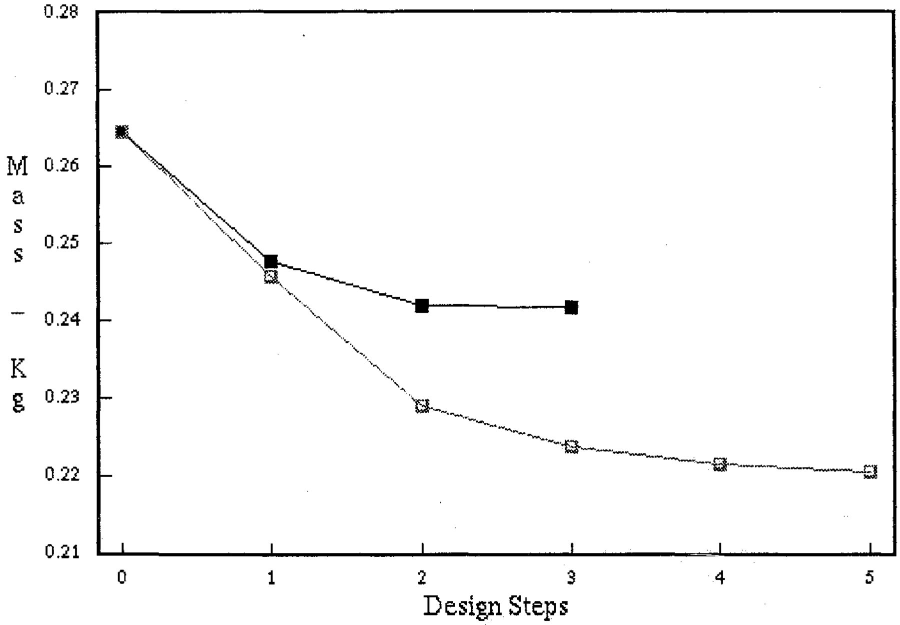


Fig. 6 Design history of brake bracket: —■—, buckling; and —⊟—, stress only.

eigenvalue of 1.0 will be approached. The optimization problem is, then, to maintain the eigenvalue λ_1 at above 1.0. Move limits used for this problem were 0.7.

Figures 3 and 4 show the results of the two optimization cases. Figure 3 displays the final shapes, along with the stress results. Figure 4 shows the design histories. The curve labeled stresses only is the design if it is assumed that buckling would not occur. In the converged design, several of the elements around the slot are at the maximum stress of 80,000 N/cm^2. The eigenvalue of the converged stress design is 0.947, indicating that this design would buckle. The curve labeled buckling has, in addition to the stress constraints, a buckling constraint of 1.0. The converged design does have a buckling value of 1.0, as well as stress constraints at 80,000 N/cm^2. A mass penalty is incurred, however, to maintain the no-buckling constraints. Both cases converged smoothly in four steps.

Brake Bracket

Analysis Problem

The part shown in Fig. 5 is a parametric model of a stamped component used for attachment of a brake pedal. The part is fixed along the vertical edge as indicated in the figure. The part is 0.3 cm thick and loaded as shown in the figure. For this study, the stresses are assumed to be limited to 7500 N/cm^2.

Although the loads are in the plane of the slot, the part will buckle out of the plane when the compressive loads become critical. Buckling occurs in the bottom of the slot because the combination of loads causes compressive stresses in that segment. The buckling factor λ_1 is 1.16 for the initial design.

Design Problem

The design problem is to minimize the mass of the part, which has three design variables, R_1, R_2, and R_3, as shown in Fig. 5. The linear elastic stresses are to be maintained at no more than 7500 N/cm^2, and the part should not be allowed to buckle. The initial design (2., 2., 2.) has an eigenvalue of 1.16, which means that buckling will not occur. An eigenvalue of 1.0 or less indicates that buckling will occur. As the mass of the part is decreased by increasing the size of the slot, the critical buckling eigenvalue of 1.0 will be approached. The optimization problem is, then, to maintain the eigenvalue to above 1.0. Move limits used for this problem were 0.7.

Figure 6 shows the results of two optimization runs. The curve labeled stresses only is the design assuming buckling would not occur. In the converged design, several of the elements around the slot are at the maximum stress of 7500 N/cm^2. The eigenvalue of the converged stress design is 0.799, indicating that this design would buckle. The curve labeled buckling has, in addition to the stress constraints, a buckling constraint of 1.0. The converged design does have a buckling value of 1.0. A mass penalty is incurred, however, to maintain the no-buckling constraints. A significant difference also exists in the number of analyses required for convergence: Whereas the buckling problem converges in three steps, the stress-only case requires five steps.

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